Fuzzy Logic Acquisition with Multi-modal Contexts

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Formal Logic

Pros:	Cons:
• Precise • Explainable • Guarantees	

Formal Logic has been fundamental as a tool in AI.

Formal Logic

Pros:	Cons:
• Precise • Explainable • Guarantees	• Grounding Problem

Grounding problem: Hard to link symbols to real-world meaning.

Formal Logic

Pros:	Cons:
• Precise • Explainable • Guarantees	• Grounding Problem • Brittleness

Brittleness: binary truth values make formal logic prone to failure under even small errors.

Formal Logic

Pros:	Cons:
• Precise • Explainable • Guarantees	• Grounding Problem • Brittleness

Consider
The problem

Task: add a microwave to a kitchen.

Constraints: $below(microwave, cupboard) \wedge leftof(microwave, oven)$

Add a Microwave to the Kitchen

$$below(microwave, cupboard) \\ \wedge\ leftof(microwave, oven)$$

Where do you place the microwave?



According to the logic, anywhere in this blue space is satisfying.

$$below(microwave, cupboard) \wedge leftof(microwave, oven)$$



Brittleness

- Prone to bad placement unless very tightly constrained.
- Microwave can be 1mm left of oven, and *leftof* is still true.

According to the logic, anywhere in this blue space is satisfying.

$$below(microwave, cupboard) \wedge leftof(microwave, oven)$$



Brittleness

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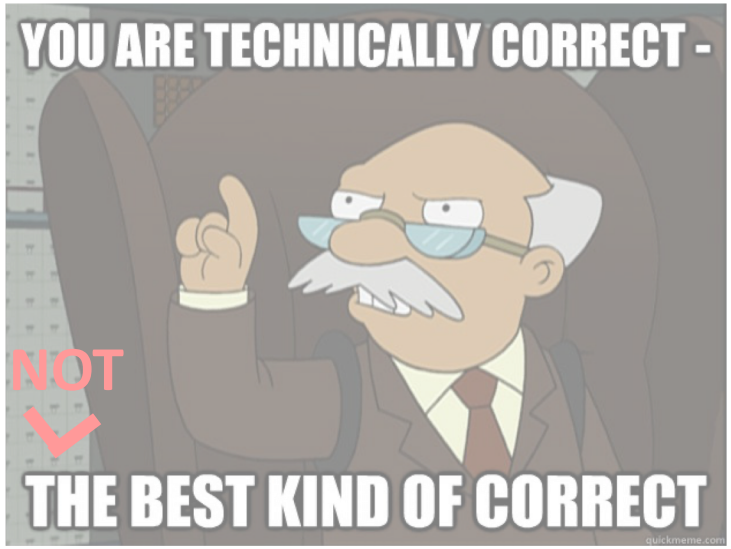
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According to the logic, anywhere in this blue space is satisfying.

$$below(microwave, cupboard) \wedge leftof(microwave, oven)$$



Grounding Problem

- Anywhere in blue is **equally** satisfying.
- Microwave on fridge is equally satisfying to one on the counter.
- This is not realistic → Not grounded in image.

Fuzzy Logic with Multi-Modal Contexts

Goal: Extend logic to address…

- Brittleness → by making the logic fuzzy.
- Grounding → by tying the logic to multi-modal contexts using ML.

Heatmap is generated by this logic.

$$below(microwave, cupboard; C) \wedge leftof(microwave, oven; C)$$



Fuzzy Logic with Multi-Modal Contexts

$$below(microwave, cupboard) \wedge leftof(microwave, oven)$$

Generate truth score based on logic and context of image.



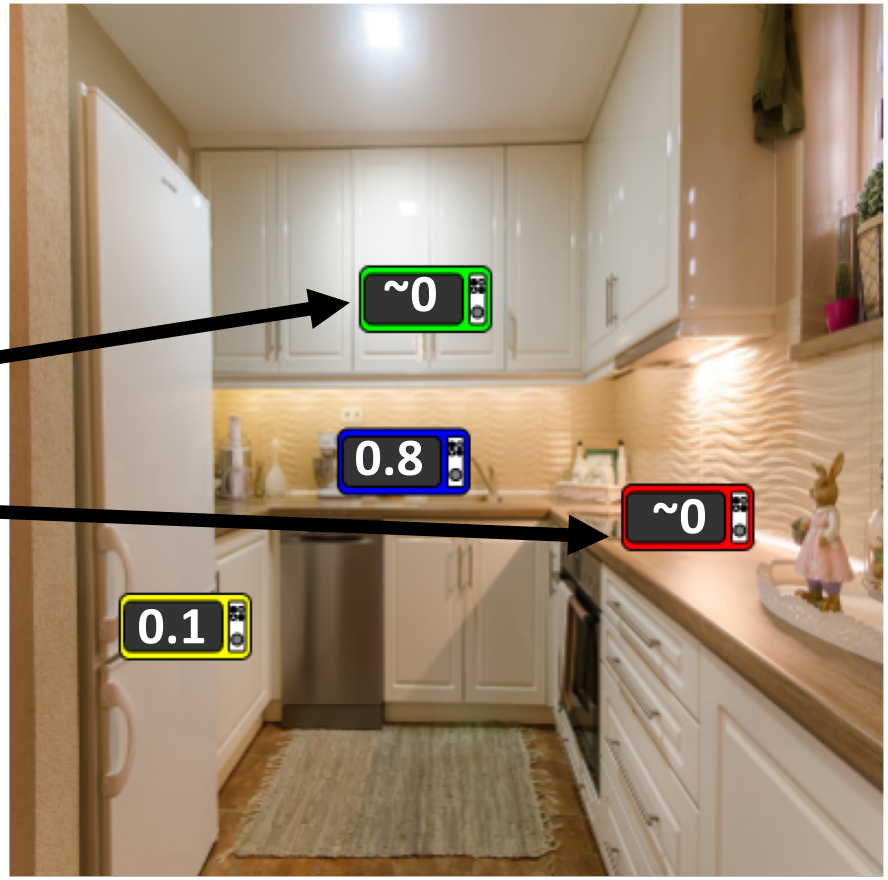
Fuzzy Logic with Multi-Modal Contexts

$$below(microwave, cupboard) \wedge leftof(microwave, oven)$$

Very low score

Not below cupboard

Not left of oven



Fuzzy Logic with Multi-Modal Contexts

$$below(microwave, cupboard) \wedge leftof(microwave, oven)$$

Low score.
Left of oven and below cupboards,
but colliding with fridge.



Fuzzy Logic with Multi-Modal Contexts

$$below(microwave, cupboard) \wedge leftof(microwave, oven)$$

Very high score.
On counter, below cupboards, and left of oven.



Fuzzy Logic with Multi-Modal Contexts

$$below(microwave, cupboard) \wedge leftof(microwave, oven)$$

Heatmap matches.



Connection to Constraint Acquisition

Constraint acquisition → uses data to **learn logic** when **meaning is known** [1, 2].

Fuzzy Logic with Multi-Modal Contexts → uses data to **learn meaning** when **logic is known**.

E.g. fuzzy meaning of *leftof* and *below*.

[1] Bessiere, C., Coletta, R., Freuder, E. C., & O'Sullivan, B. (2004). Leveraging the learning power of examples in automated constraint acquisition. In M. Wallace (Ed.), CP 2004 (pp. 123–137). Springer.
[2] Prestwich, S. D., Freuder, E. C., O'Sullivan, B., & Browne, D. (2021). Classifier-based constraint acquisition. Annals of Mathematics and Artificial Intelligence, 89(7), 655–674.

Intro & Example

Logic Definition

Learning & Inference

Symbols

- Constants
  - E.g. rectangle (x,y,w,h) of existing objects.
  - Here, a **washing machine**.
- Variables
  - E.g. The rectangle of the microwave we want to add.
  - Variables have domains.



Symbols

- Constants: existing objects.
- Variables: new objects.
- Contexts
  - Image, text, video, etc.
  - E.g. this image.



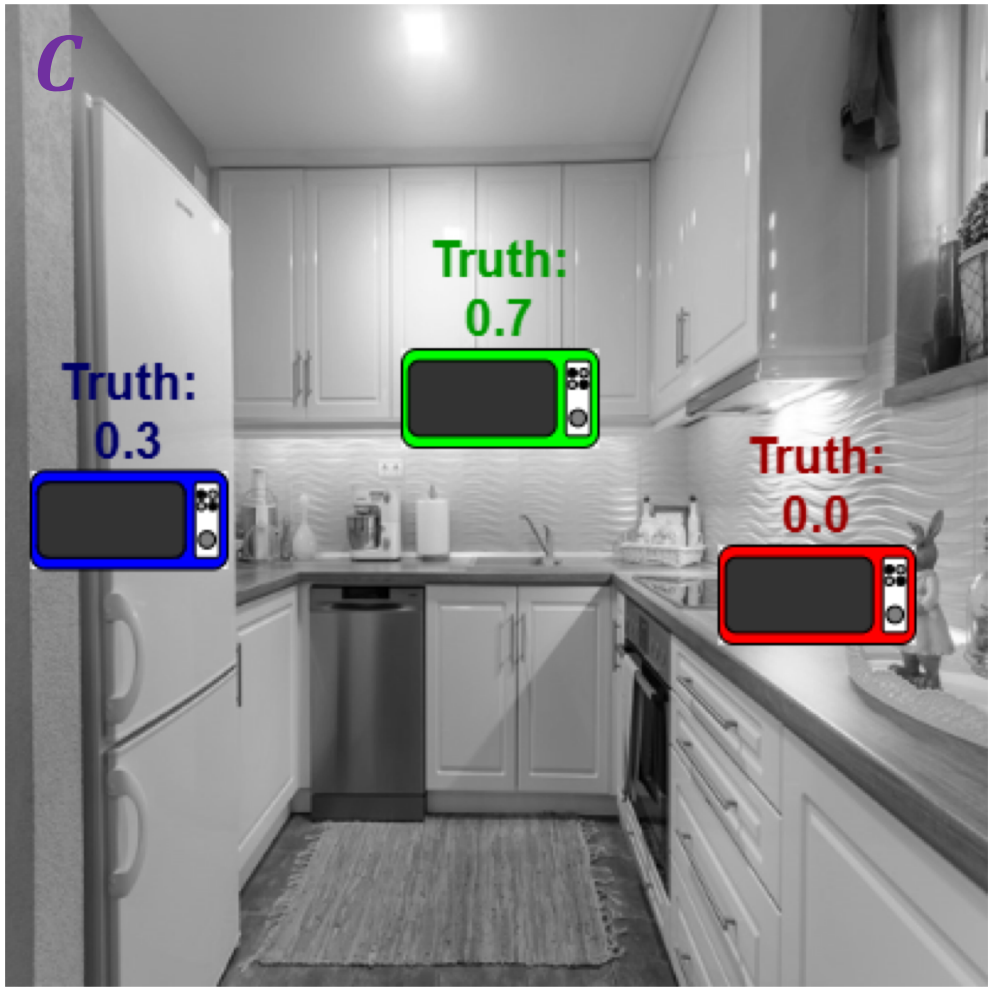
Predicates

- leftof, rightof, above, below, etc.
- Input are variables, constants, contexts.
  - Here, rectangles with (x,y,w,h) components.
- Returns: fuzzy truth value.
  - degree of truth given context.
  - 0 to 1 range.

Predicates

$$leftof(microwave, oven; C)$$

- Red: not left of oven → not true.
- Blue: left, but too far → low truth.
- Green: good place → high truth.



Connectives

Conjunction, disjunction, and negation.

Connective Operators

$$X \wedge Y = \min(X, Y)$$

$$X \vee Y = \max(X, Y)$$

$$\neg X = 1 - X$$

(X and Y are predicates)

Intuition

Least true is representative.

Most true is representative.

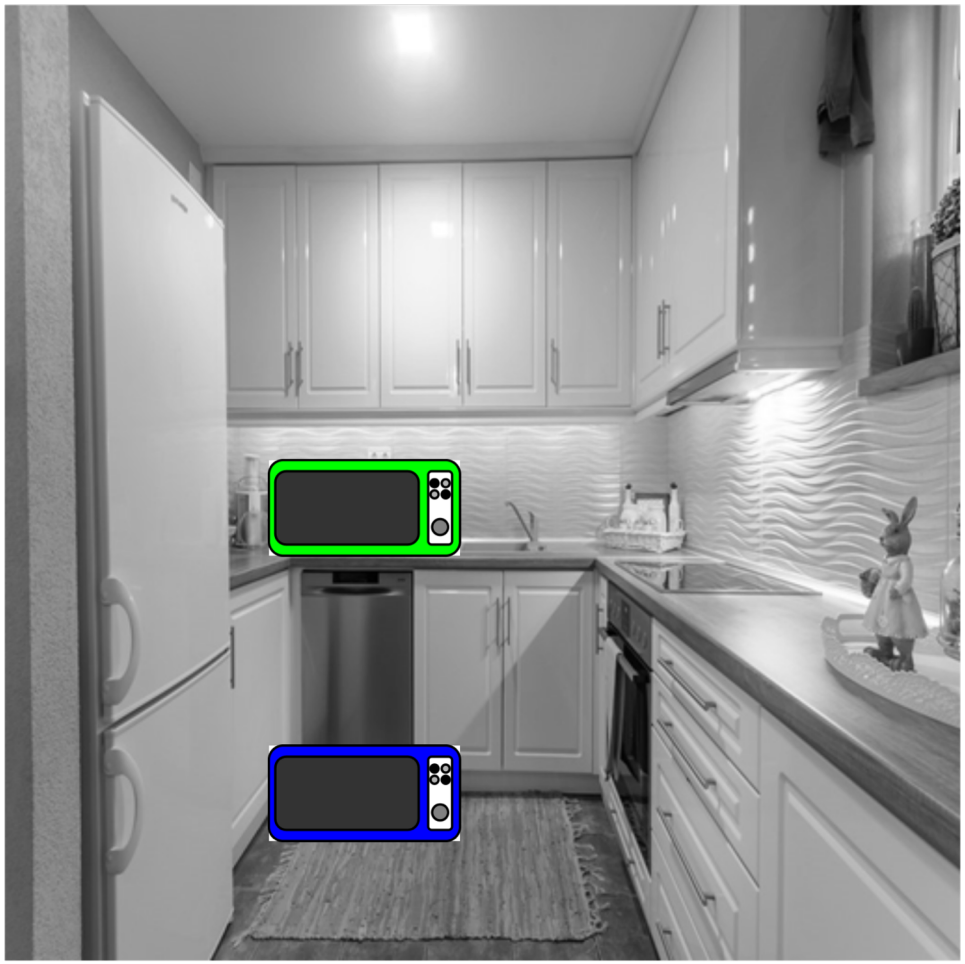
Invert truth.

Common formulation in fuzzy logic [cite].

Connectives

$$below(microwave, cupboard) \wedge leftof(microwave, oven)$$

color	leftof	below	leftof ∧ below
Green	0.9	0.8	**0.8**
Blue	0.9	0.3	0.3



$$X \wedge Y = \min(X, Y)$$

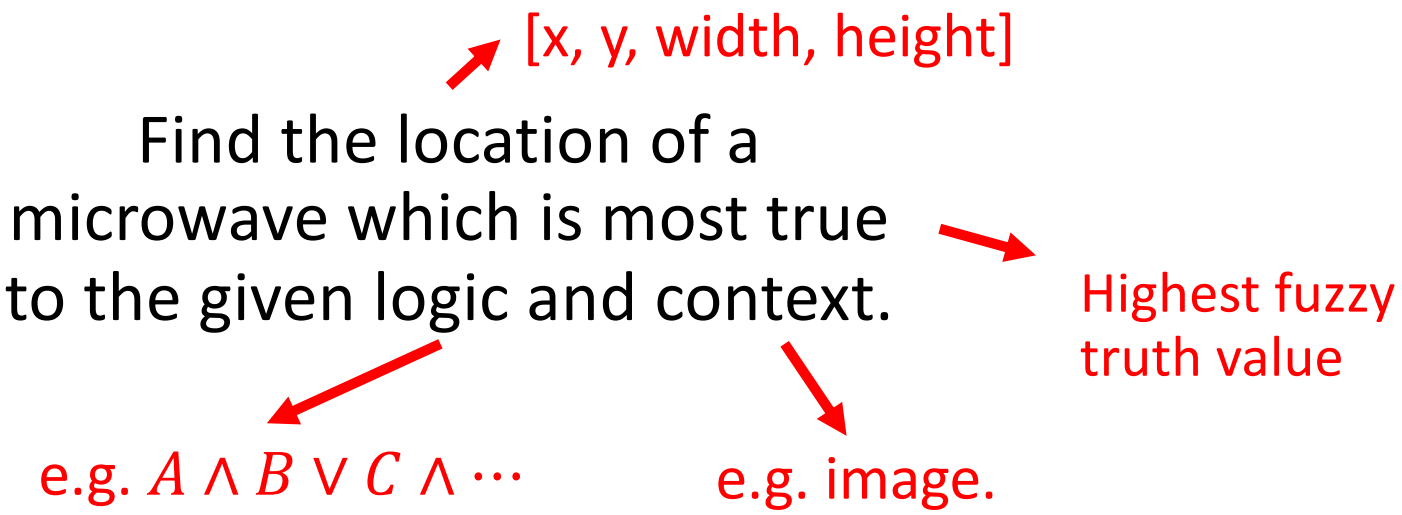
Intro & Example

Logic Definition

Learning & Inference

Learning & Inference

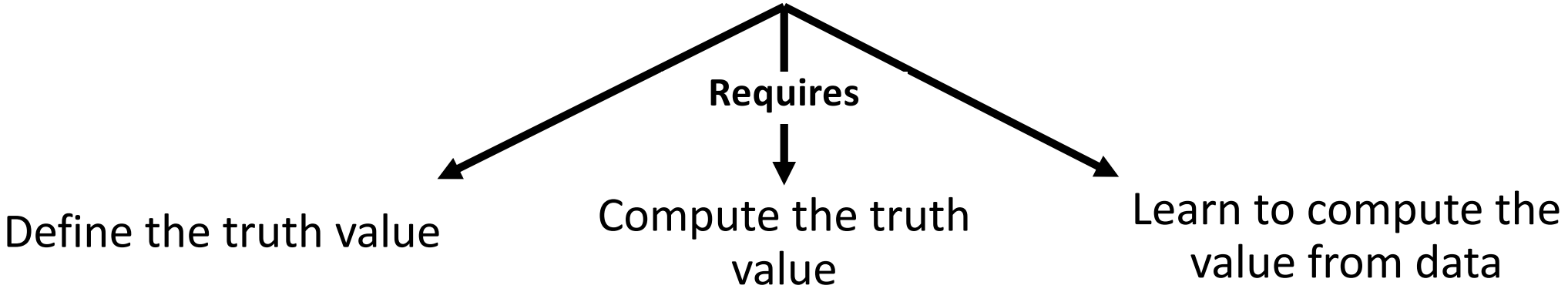
Let's say we want to solve a problem like:



Learning & Inference

Let’s say we want to solve a problem like:

Find the location of a microwave which is most true to the given logic and context.

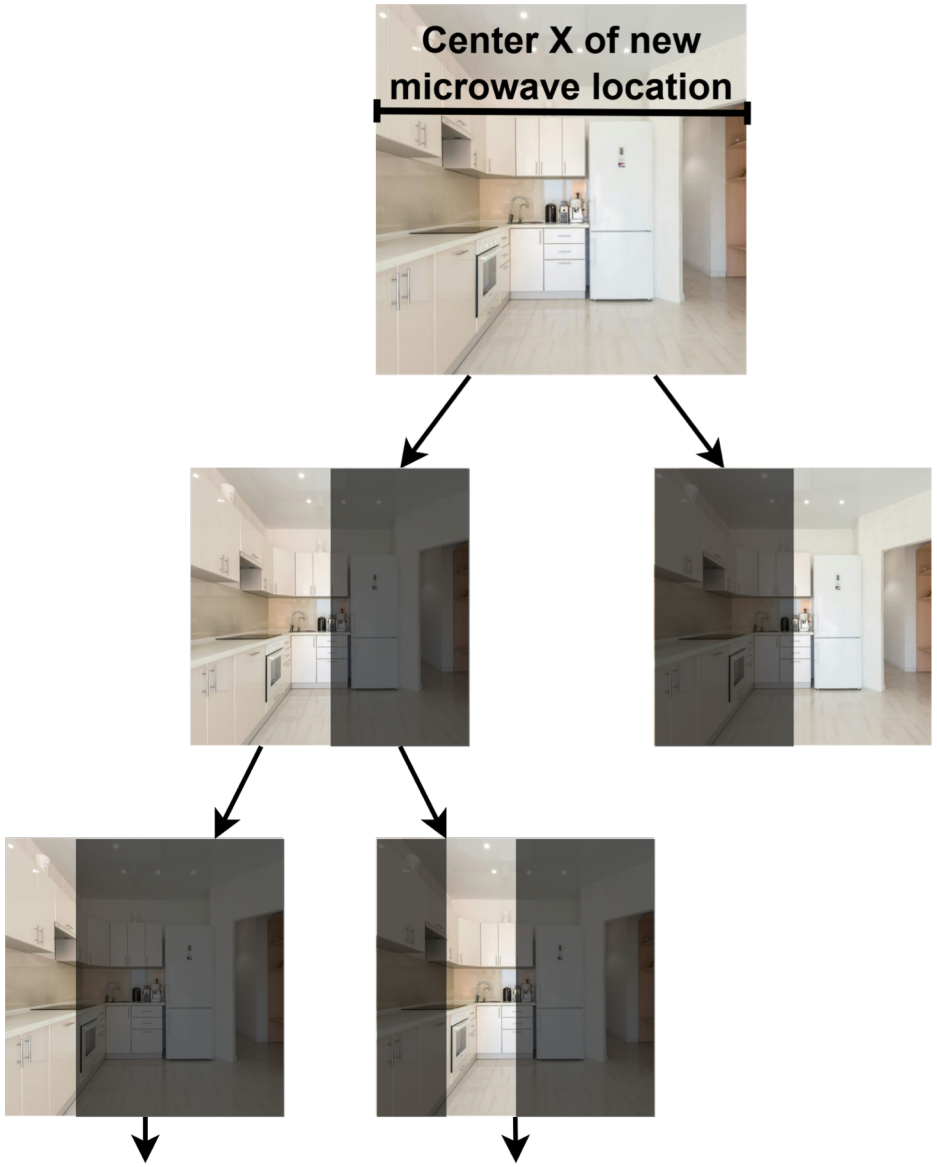


Define the truth value

- Define fuzzy truth value as *probability* of seeing the assignment given context.
- This means the most probable assignment, conditioned on the context, has the maximum truth.

Compute truth value

- One way to compute this value.
- Define a tree for each predicate over the domain of each variable.
- Each node is a sub-domain.
- Each leaf is a single solution.
  - E.g. a chosen bounding box rectangle.



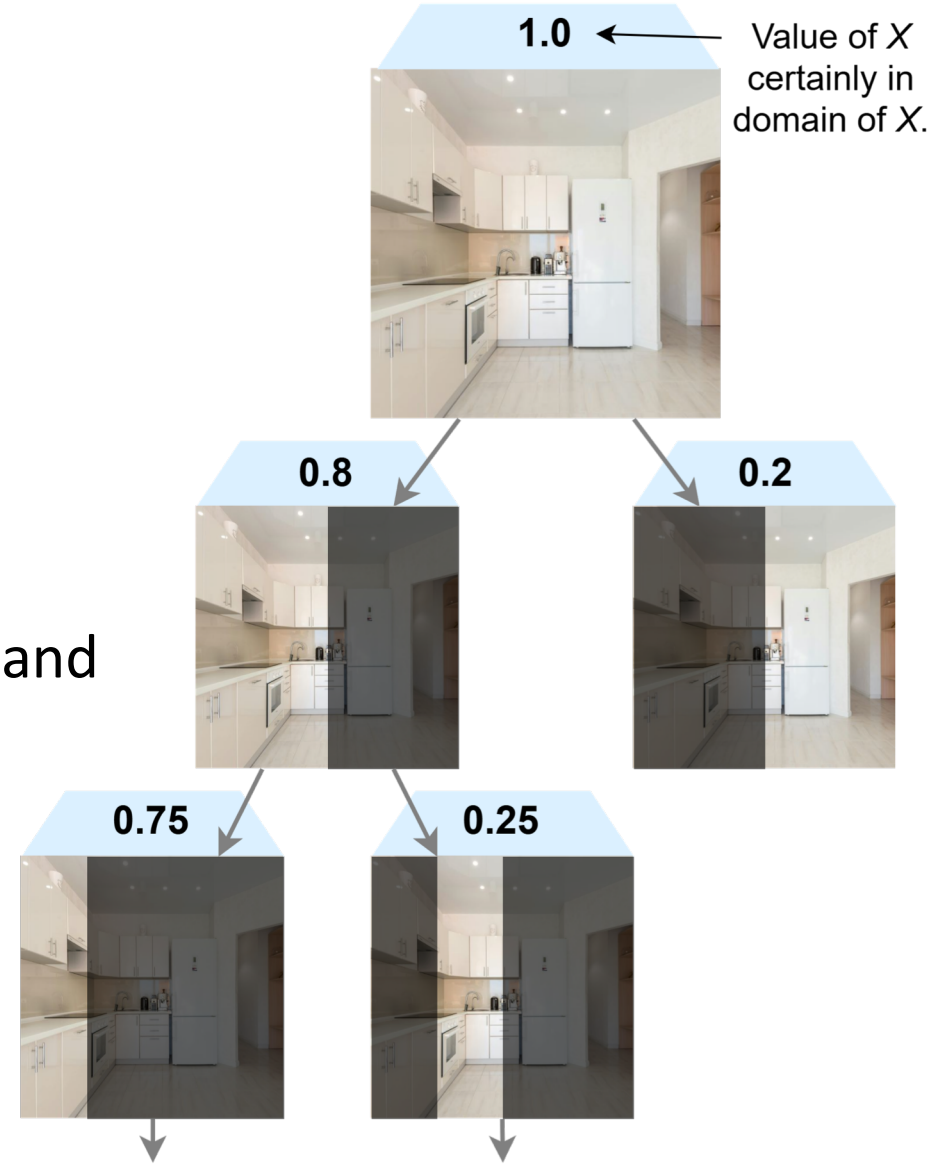
Compute truth value

- Define a tree for each predicate over the domain of each variable.
- Each node is a sub-domain.
  - **A subset of microwave locations.**
- Each leaf is a single solution.
  - **One specific microwave location.**



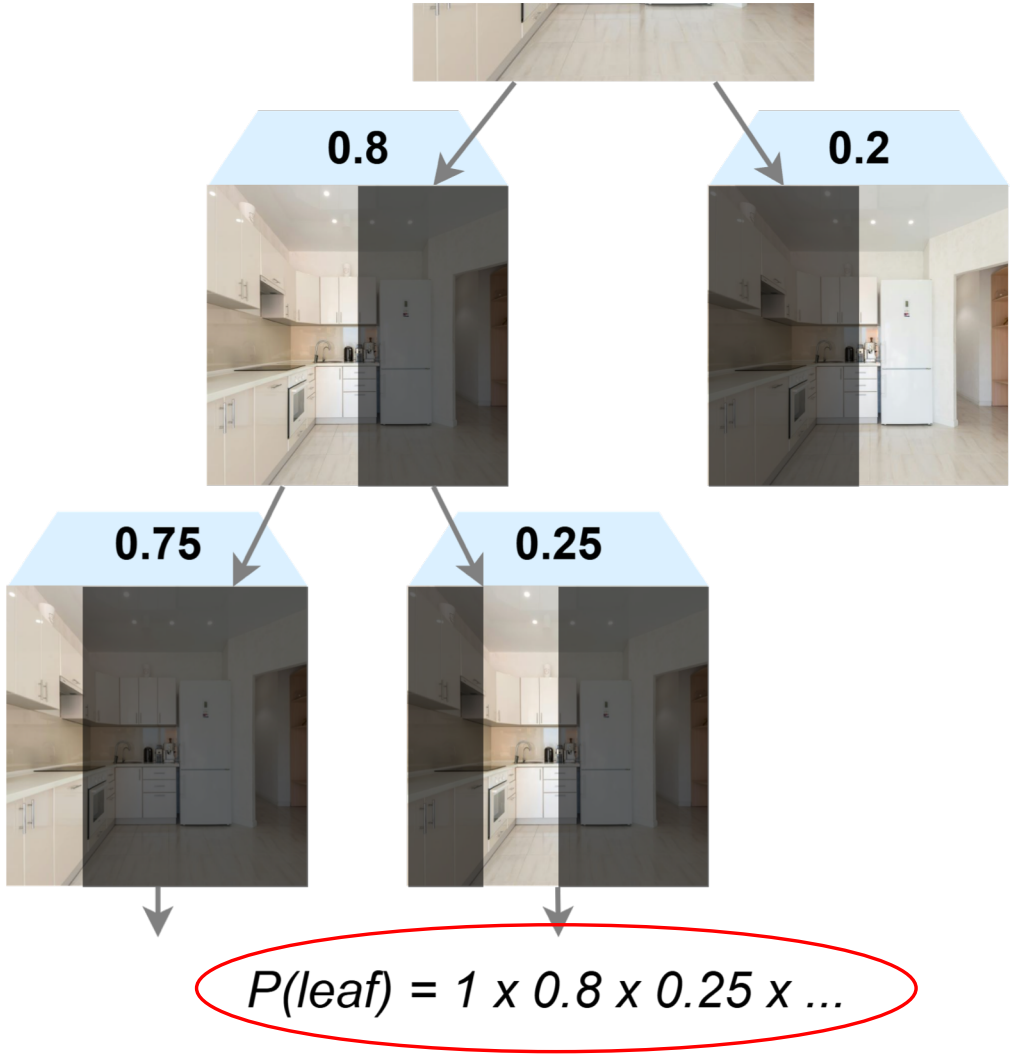
Compute truth value

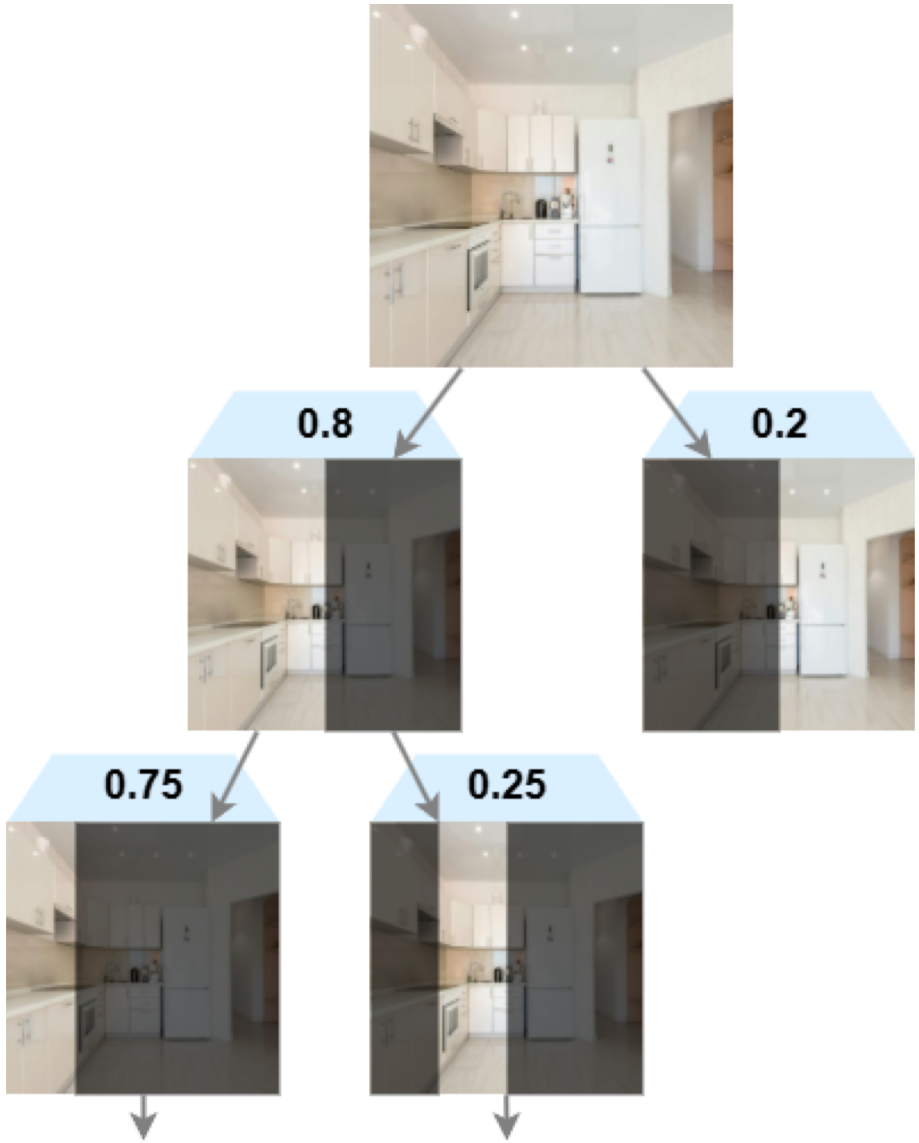
- Define a tree for each predicate over the domain of each variable.
- Each node is a sub-domain.
- Each leaf is a single solution.
- Each child has a probability given its parent and the context.
  - E.g. 80% chance of seeing left side after root.



Compute truth value

- Define a tree for each predicate over the domain of each variable.
- Each node is a sub-domain.
- Each leaf is a single solution.
- Each child has a probability given its parent and the context.
- Leaf node probabilities are product of probabilities from root to leaf.

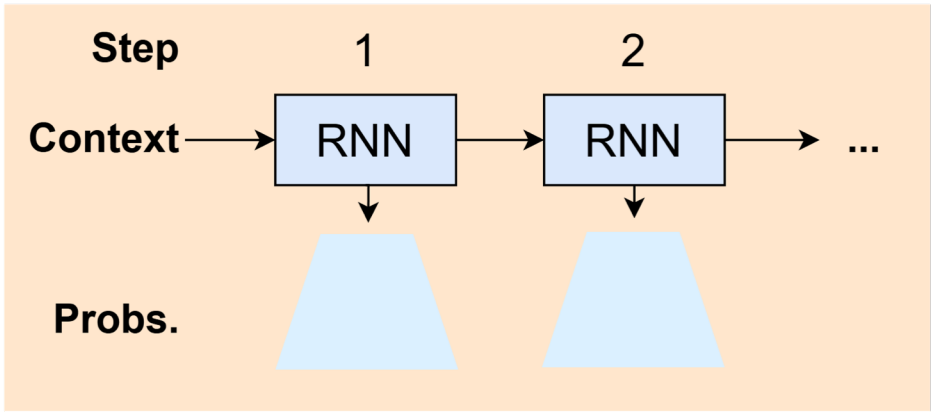


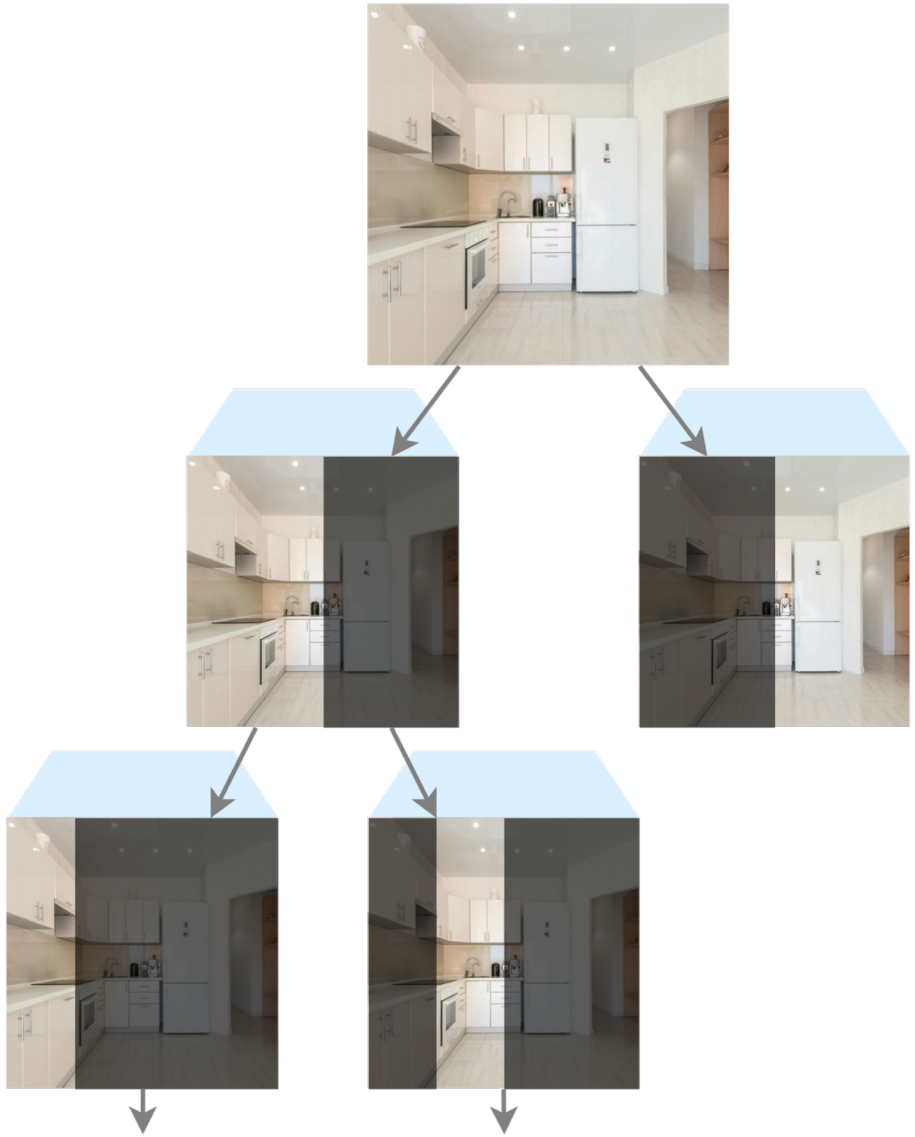


P(leaf) = 1 x 0.8 x 0.25 x ...

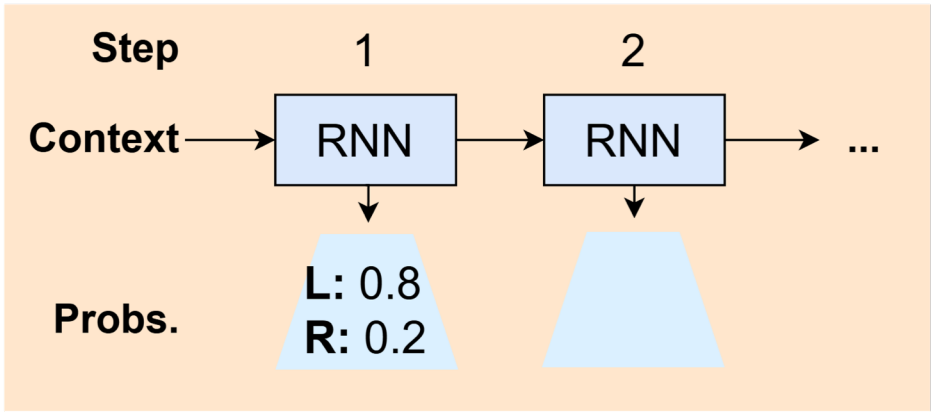
RNN to Learn From Data

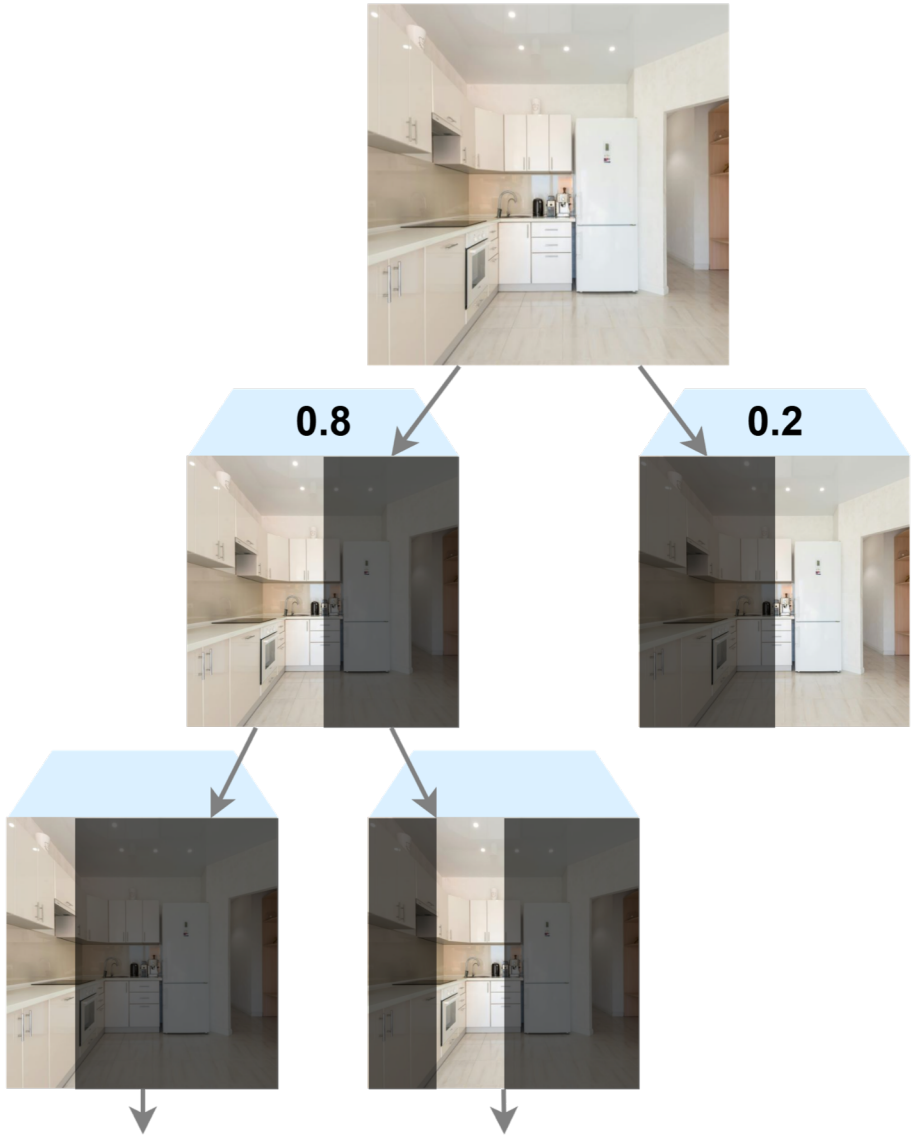
Recurrent Neural Network (RNN)



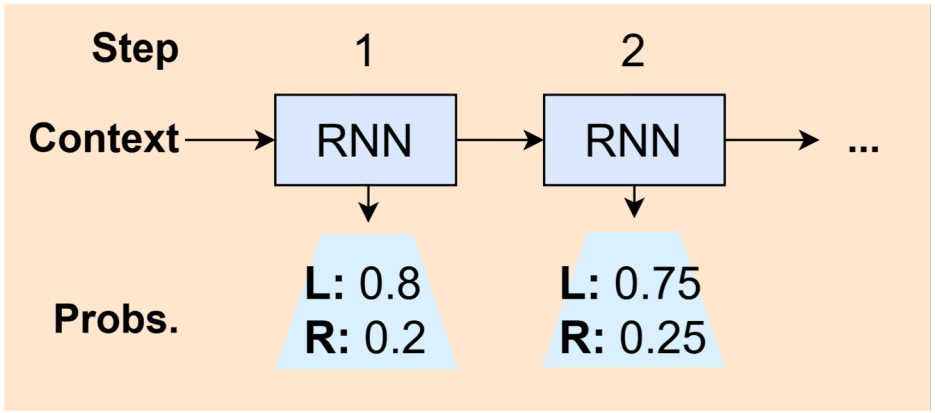


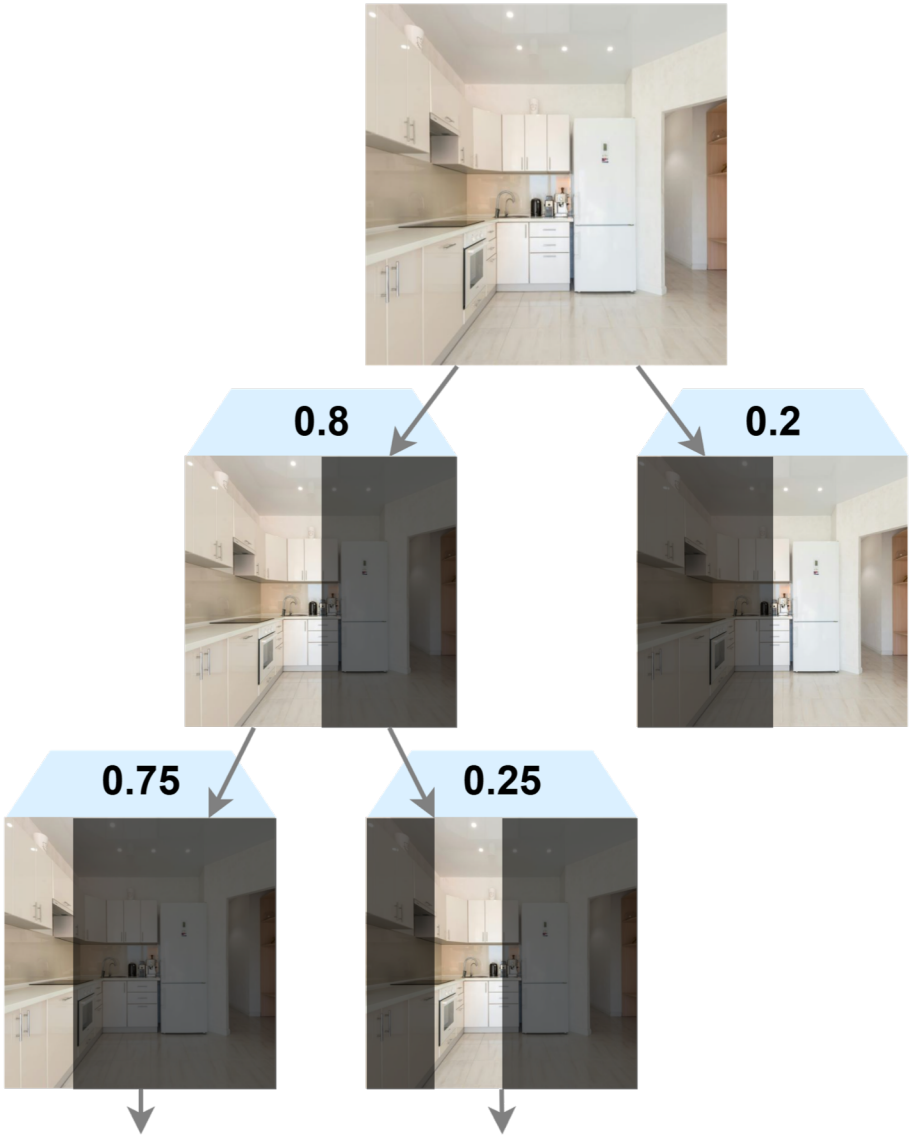
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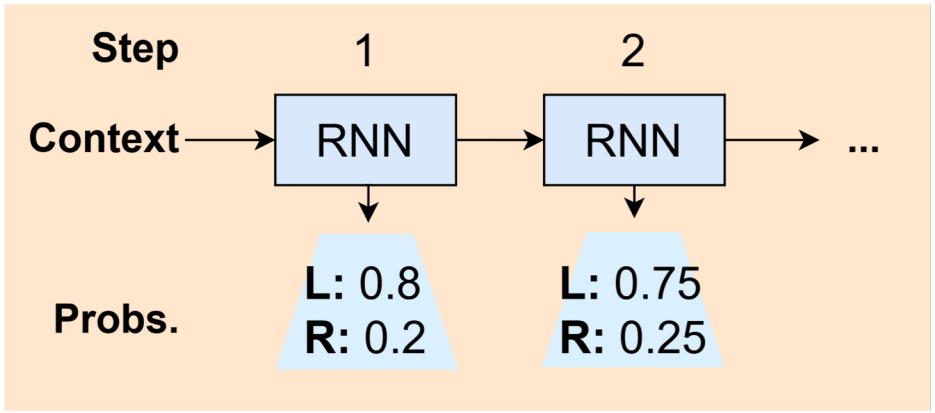


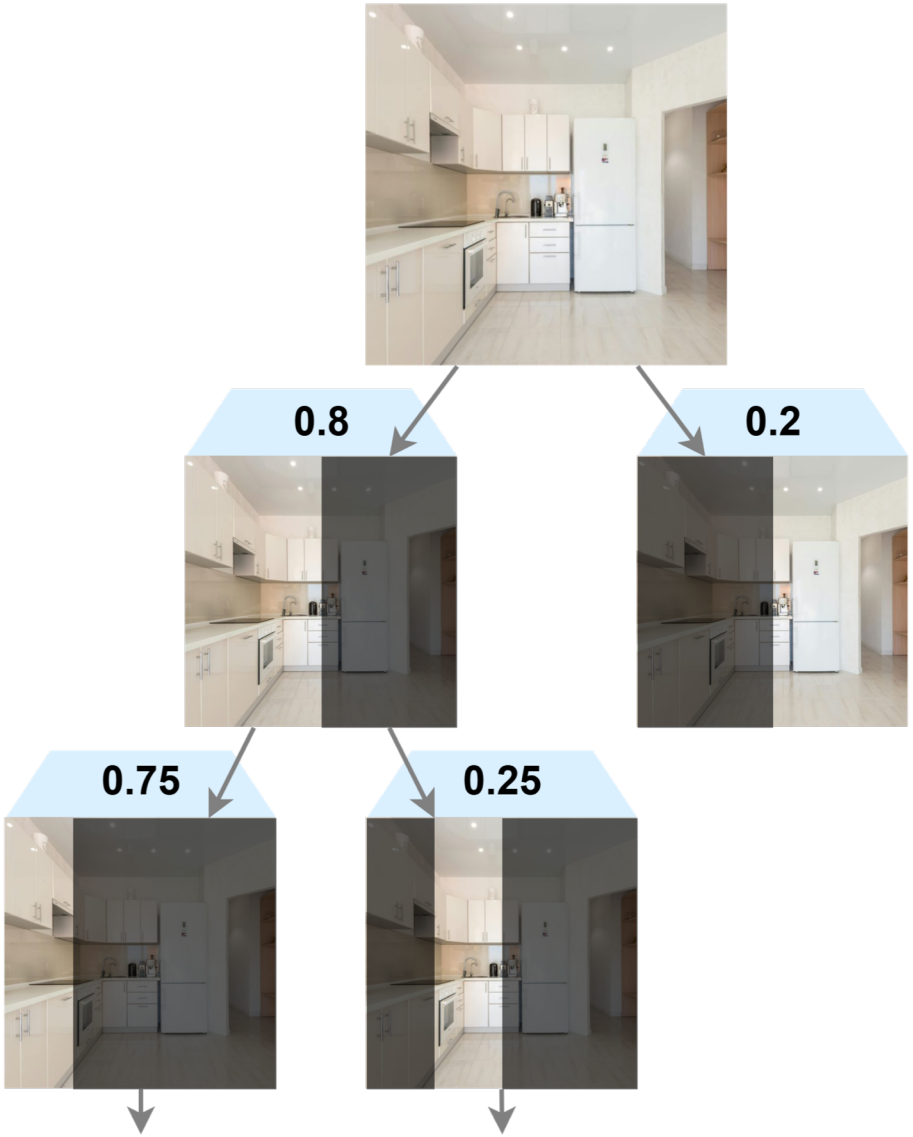
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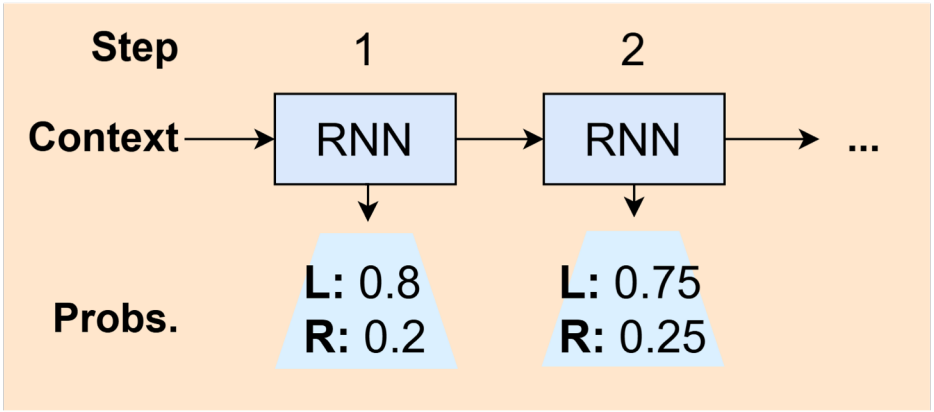
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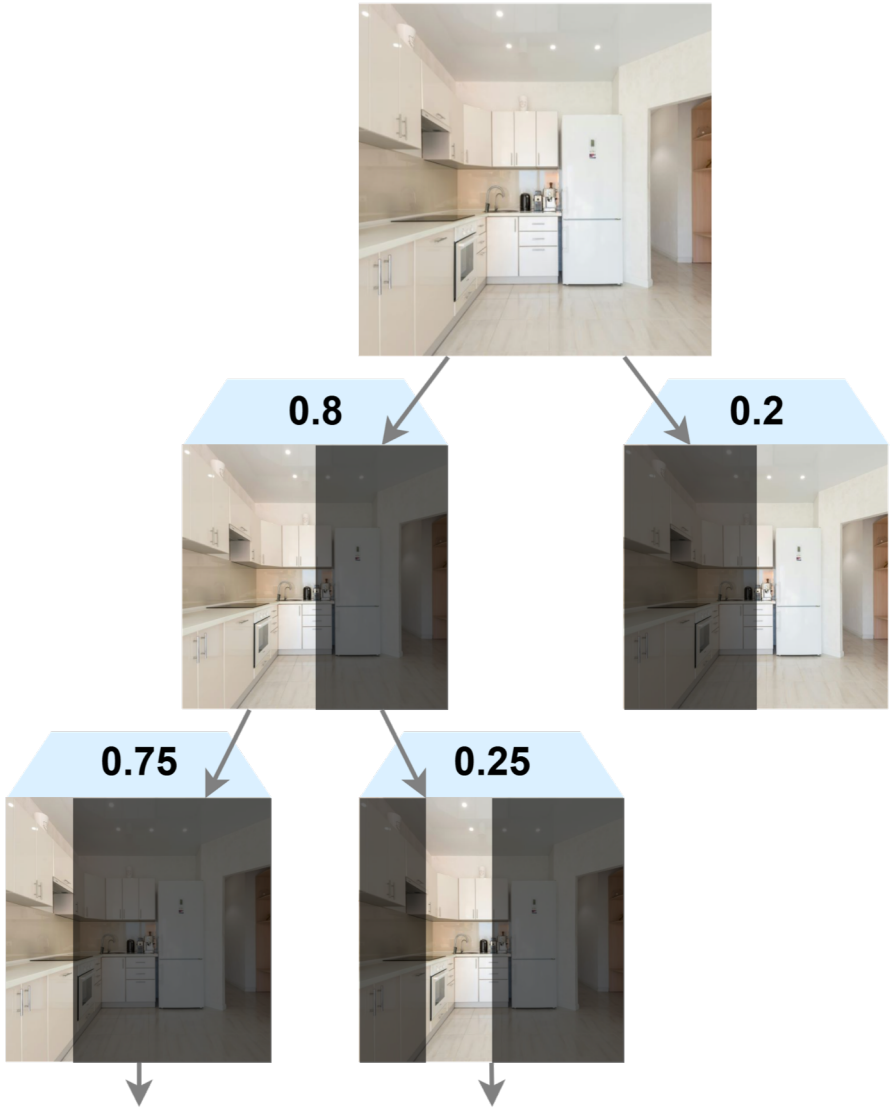


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RNN to Learn From Data



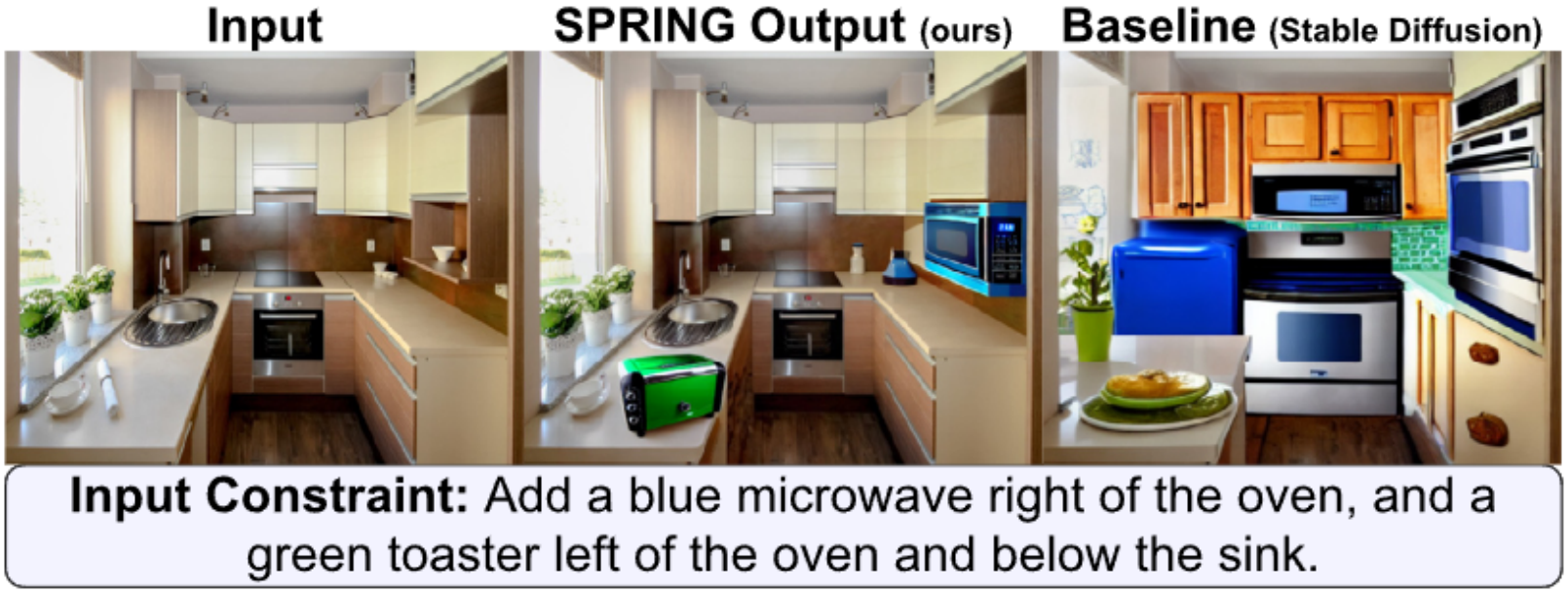
$$P(leaf) = 1 \times 0.8 \times 0.25 \times \ldots$$



RNN like this can be trained with supervised approaches [5].

[5] Ronald J. Williams and David Zipser. "A Learning Algorithm for Continually Running Fully Recurrent Neural Networks." Neural Computation, 1989

Spatial **R**easoning **In**tegrated **G**enerator



- Image generation task requires implicit preferences and explicit constraints.
- This is our prior work. We hope to apply our fuzzy logic with multi-modal contexts to improve this work.

Come see our journal track talk (12:05pm, Room 121C) Friday!

Published in AIJ [6].

Conclusion

- We propose a new fuzzy logic to handle multi-modal contexts.
- Addresses:
  - Brittleness → by making the logic fuzzy.
  - Grounding → by tying the logic to multi-modal contexts.
- Uses RNN and tree structure to do learning and inference on this logic.

References

1. Bessiere, C., Coletta, R., Freuder, E. C., & O'Sullivan, B. (2004). Leveraging the learning power of examples in automated constraint acquisition. In M. Wallace (Ed.), CP 2004 (pp. 123–137). Springer.
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Thank you!



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